GEORGIA INSTITUTE OF TECHNOLOGY — OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

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Project Director: Dr. D. T. Paris — School/Lab EE

Sponsor: Naval Coastal Systems Center, Panama City, FL

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Title: Development and Implementation of a Computer Model to Predict IDP Bubble Population

ADMINISTRATIVE DATA

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RESTRICTIONS

See Attached Government Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval — Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with Government

COMMENTS:

Subcontract to the University of Georgia

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SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 8-25-1987

Project No. E-21-124 School/~~Lab~~ EE

Includes Subproject No.(s) N/A

Project Director(s) Dr. D. T. Paris GTRC / ~~GIT~~

Sponsor Naval Coastal Systems Center, Panama City, Fl

Title Development and Implementation of a Computer Model to Predict IDP Bubble Population

Effective Completion Date: 8-17-1987 (Performance) 8-17-1987 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [x] Final Invoice or Final Fiscal Report
- [x] Closing Documents
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- [x] Govt. Property Inventory & Related Certificate
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R. Embry
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FORM OCA 69.285

Progress Report

PREDICTION OF INITIAL DATA PLANE BUBBLE POPULATIONS"

I. INTRODUCTION

The objective of the project, as stated in the statement of work "... is to develop and implement a computer model to predict IDP bubble populations given the size and speed of the surface ship.". As input the Government was to provide the near ship hydrodynamics and the results of measurements made on wake bubble populations with the RV/Athens. But for reasons beyond the contractor's control this data has not been available. Hence the contractor has proceeded to analyze the near ship bubble transport and source problem considering that the needed data could be input in principle. To this end the contractor has reviewed the literature on near ship and particularly near stern ship hydrodynamics and surface ship and sea state bubble populations. This review has demonstrated to the contractor the necessity of modifying the bubble evolution equation to include the special effects of near ship hydrodynamics. The bubble population can then be predicted on the desired downstream IDP.

II. Summary of Technical Status and Work Performed

The major portion of the initial work of this contract has been the literature review on near hull hydrodynamics and bubble populations, the determination of the modifications of the bubble evolution equation and the best proceedure with which to apply it, and the characterization of the initial and sea state bubble distributions.

A. The Sources

The literature review indicates the existence of 3 possible bubble sources for bubbles at the current IDP. The first and most visible is that of the white water wake in the immediate area of the stern. Here the literature indicates for the hulls of interest a region of flow boundary layer separation near a ship's stern. The associated zone of "attached" water has a high air content and provides a "white water" surface for the nearly streamlined flow below. The capture mcechanism is the turbulence associated with the mixing region at the interface and downstream propeller swirl. This together with the breaking transverse waves of the white water wake constitute one source for the wake bubble density.

Other measurements demonstrate the existance of an ambient population whose bubble densities are on the order of the deep wake bubble population. The ambient population together with air entrained along the hull provides a second highly probable source for deep wake bubbles. The capture mechanism here is propeller suction and stern or bilge vortices associated with flow past the hull. This mechanism would explain the extent of the deep wake width (the full beam) at the stern location.

Finally one has breaking bow and stern waves as possible sources, and also those which combine in a random manner with the chop of the ambient sea state. However this bubble source is probably not significantly associated with the deep wake in the near ship region beyond being viewed as possibly creating an effective change in the ambient population away from the ship's hull and creating changes in the hydrodynamic flow. In contrast to the breaking waves of a sea state the wind velocity is typically in opposition to the direction of bow and stern wave propagation.

Figures I, II, and III illustrate possible near stern flows contributing to the growth of the bubble wake into its stern cross-sectional profile. It seems reasonable to specify the bubble density along a wake cross-section located at the stern and cut along the boundary of the hull boundary layer and subsequent separation zone. The sea surface and the downstream boundary of the separated boundary layer and the ocean surface then form the remainder of the surface on which the initial bubble density need be specified. Specification of the bubble density along these surfaces then provides the initial data required for a solution to the bubble density evolution equation.

B. Modification of the Bubble Evolution Equation

As will be seen in the next section, the bubble evolution equation has the steady state form

$$(1) \qquad \vec{\nabla}\cdot\{\vec{u}\,\Psi + \vec{J}\} + (g\Psi)_r = 0,$$

where $\vec{u}$ is the average bubble velocity, Ψ the bubble density , g the bubble disolution rate, and

$$(2) \qquad J^{l} = G^{lm}\nabla_m\Psi + D^{lm}\nabla_m\Psi .$$

$\vec{J}$ characterizes the effect of velocity fluctuations from the average. The tensors D^{lm} and G^{lm} characterize "diffusion" associated with deviations from the average due to conventional diffusion with vorticity. Their specific forms are given below.

Using Green's theorem it is not difficult to see that the solution to the boundary value problem is specified by the integral

$$(3) \qquad \Psi(\vec{x};\vec{r}) = \int_S ds^j \int dr' \{\phi u^j \Psi + \phi G^{jk}\nabla_k\Psi + \Psi G^{jk}\nabla_k\phi - \phi D^{jk}\nabla_k\Psi + \Psi D^{jk}\nabla_k\phi\},$$

where S is a closed 2d spatial surface and (x,y,z) represents an interior point, and ϕ represents the Green's function belonging to (1), a linear equation.

The surface S can be identified with that discussed in the last section and illustrated in Figure IV if there are no very deep bubbles and sufficiently far from the wake axis ambient bubble transport into the wake can be neglected.

An alternate formalism is to place a cross plane ahead of the ship and treat the remaining part of S as the sea surface and the boundary of the boundary layer running along the ships hull and the separation zone. This latter situation is illustrated in Figure V.

B1. The Bubble Diffusion Current Density

The expression for the conventional and "vortical" diffusion follow from viewing the fluid as a "random walk". Each probable fluid trajectory bringing bubbles to a spacial point x is associated with a probability p_n and a velocity $\vec{v}_n$ and belongs to the evolution equation

$$\psi^{(n)}_t + \vec{\nabla} \cdot \{ \vec{v}_n \psi^{(n)} \} + (q\psi^{(n)})_r = 0 \qquad (4)$$

Letting (x_n, r_n) denote the start point of the nth trajectory and using the method of characteristics for each solution to (4), one finds from averaging over n that

$$\nabla_\ell \{ u^\ell \psi + \Sigma p_n \Delta v_n^\ell e^{-I(\vec{x}, \vec{x}_n)} (\psi(\vec{x}_n; r_n) - \psi(\vec{x}; r)) + (q\psi)_r = 0, \qquad (5)$$

where

$$\vec{v}_n = \vec{u} + \Delta\vec{v}_n,$$

where $\vec{u}$ is the average velocity $\Sigma p_n \vec{v}_n$.

In the coordinate system of the ship where away from the boundary layers the mean flow clearly dominates one can associate the change in bubble radius along each trajectory with that on the mean trajectory. Letting

$$\Delta \vec{s}_n = \vec{x}_n - \vec{x}, \qquad I(\vec{x}, \bar{x}_n) = I$$

consistent with the previous sentence, one finds to 1st order the steady state equation

$$\nabla_\ell \{ u^\ell \psi + e^{-I} (\sum_n p_n \Delta v_n^\ell \Delta s_n^m) \nabla_m \psi + (q\psi)_r = 0. \qquad (6)$$

Comparing this equation with (1) and (2) one finds that

$$D^{\ell m} = e^{-I} \sum_n p_n \Delta v_n^{(\ell} \Delta s_n^{m)}, \quad G^{\ell m} = e^{-I} \sum_n p_n \Delta v_n^{[\ell} \Delta s_n^{m]}, \qquad (7)$$

where ( ) and [ ] bracketing the exponents denotes tensor symmetrization and antisymmetrization.

B2. Identification of Diffusion and Vortical Diffusion Terms

The vortical and ordinary diffusion are related to the flow parameters through identification with the physical dynamics of the flow. For example, from (7) one readily sees from the definition of angular momentum that

(8) $$G^{\ell m} = e^{-I}\,\Delta L^{\ell m}/\rho \,,$$

where $\Delta L^{\ell m}$ is the average change in the angular momentum of the fluid elements reaching $\vec{x}$ relative to the average flow velocity during the correlation time and ρ is the density of the fluid. One has the change in angular momentum here because each fluid element has by definition instantaneous angular momentum zero relative to the point $\vec{x}$.

But the relative angular momentum change is related to the torque generated by the Reynolds stress by

(9) $$\Delta L^{\ell m} = 2t\,\Delta s^{[\ell}\,\nabla_p\,\tau^{m]p}$$

where

$$\Delta\vec{s} = \sum \rho_n\,\Delta\vec{s}_n .$$

Hence using (9) and (8) one finds

(10) $$G^{\ell m} = e^{-I}\,\frac{t}{\rho}\,\Delta s^{[\ell}\,\nabla_p\,\tau^{m]p} .$$

The diffusion term can also be related to the Reynolds stress. Using (7),

(11) $$D^{\ell m} = e^{-I}\sum \rho_n\,\Delta v_n^{(\ell}\,\Delta s_n^{m)} = e^{-I}\,t\sum \rho_n\,a_n^{(\ell}\,\Delta s_n^{m)}$$

where a_n^{ℓ} is the relative average acceleration over the time t. Hence, the trace $\sum D^{\ell\ell}$ is proportional to the average work done, the change in relative kinetic energy, which in turn coincides with the trace of the Reynolds stress tensor. Hence, on the basis that the 2 tensors have the same trace we assert

(12) $$D^{\ell m} = e^{-I}\,\frac{t}{\rho}\,\Delta s^{k}\,\nabla_k\,\tau^{\ell m} .$$

From here $G^{\ell m}$ and $D^{\ell m}$ are easily related to the mean flow parameters by choosing a particular turbulence model, such as the eddy diffusion model. This has been done.

B3. The Characteristic Scaling Function

The exponent I in (7) measures the change in scale due to the change in the "volume" elements associated with the transformation of $(x,y,z;r)$ into $(x_n,y_n,z_n;r_n)$ and involves an integration along the average trajectory. Since the time scale t in the near wake region is short this integration can be approximated by a linearized transformation, and this transformation is being developed.

C. The Sea State Source Distribution

It is stated in the "Statement of Work - Technical Requirements" that the bubble sizes within a small element of volume may be approximated by a sea state distribution, a-conclusion derived from observations on oceanic bubble distributions and illustrated in Figure VI. A log-log plot of bubble density vs radius suggests such a distribution may be approximated by a maximum value, a power law increase in radius for small radii, and a power law decrease for larger radii. An analytical investigation has been carried out to determine the basis of this observation in order to determine a way to simplify the numerical integration of (1-2) or work instead with a derived equation involving only the 4 parameters.

A result based upon the fixed suspension model most clearly illustrates the "sea state" distribution concept. In this model the bubble density for bubbles of radius r at time t is given in terms of the density at time t by

$$\psi(r,t) = \psi(r_0,t_0)\,\frac{g(r_0)}{g(r)}, \tag{13}$$

where r is the radius of the bubble at time t . Thus one sees that the bubble density distribution is "modulated" by a function independent of the initial density and dependent only upon the disolution model. This is the source of the low radii behavior of the sea state bubble distribution, the large radii behavier following from the bubble creation and entrainment process.

For the short time scales existing in the near ship region this type of modeling allows the integrations over trajectories implied in the determination if I in (10) and (12) to be avoided, and also allows the distribution parameters of ambient sea states to check the disolution model.

III. - Overall Status

The modification of the bubble evolution equation has been completed to the extent that the equation is regarded as valid beyond the boundries of the boundary layers. Given the mean flow and its 1st and 2nd spatial derivatives in the near wake region (that is in the "half-volume" enclosed by the surface in (3)) equation (1) can be integrated to provide the desired IDP bubble population provided the initial data on the surface of (3) is known together with the appropriate boundary conditions.

Models for the bubble density "modulation" function have been investigated and a good "feel" has been developed for the connection between the disolution model and the parameters of the "universal sea state" distribution. A specific model for the sea state distributions associated with the 3 possible classes of sources has not been completed.

IV. Future Work

The remaining effort will be directed towards -

1. programming the modifications to the bubble diffusion equation for insertion into the NCSC bubble evolution code,

2. developing and modeling the initial bubble distribution for the integration to the IDP, and

3. developing the appropriate boundary conditions along the sea surface and the boundaries of the ship's turbulent boundary layers.

V. Required List of Parameters.

As per the contract the required list of parameters needed to characterize the bubble distribution as a function of ship class and operating conditions include:

1. the mean flow field in the near ship region,

2. the diffusion and vortical diffusion fields in the near ship region,

3. the ship boundary layer and separation zone boundaries, and

4. the bubble disolution model parameters.

VI. Budget Analysis

The contract execution was planned for approximately a 90 day period with all funds to be expended by June 12, 1987.
Approximate expenditures to date are as follows:

a.	Salary and Staff Benefits	13.6K.
b.	student	.3K.
c.	computer	.3K.
d.	travel	.7K.
e.	overhead	.6K.

Approximate man hours expended are as follows (in terms of total allocated)

a.	principle investigator	100%.
b.	student	40%.

Graphs illustrating percent expenditures for the different catagories is illustrated in Figure VII.

It is anticipated that all funds will be expended by June 12, 1987.

PREDICTION OF INITIAL DATA PLANE BUBBLE POPULATIONS

by

C. A. Uzes

in fulfillment of statement of work as per contract #

N61331-85-D-0025-0024

Abstract

The surface ship bubble boundary value and near wake evolution problems are studied. The "initial" data needed for determining bubble wake evolution is determined and the general form of the near ship bubble evolution equation investigated. Based upon the notion of a hydrodynamic correlation time and a "random walk" simulation of turbulence, the form of the bubble diffusion tensor is established and a new vorticiy based transport mechanism is derived. The latter allows turbulent capture of bubbles from a flow zone interface even when the mean flow is tangent to the interface, a vorticity and turbulence dependent mechanism. Modifications of current bubble transport codes as discussed and the procedure for solving the bubble boundary value problem is discussed.

TABLE OF CONTENTS

PREDICTION OF INITIAL DATA PLANE BUBBLE POPULATIONS"

I. Introduction

The objective of the project, as stated in the statement of work "... is to develop and implement a computer model to predict initial data plane (IDP) bubble populations given the size and speed of the surface ship.". As input the Government was to provide the near ship hydrodynamics and the results of measurements made on wake bubble populations with the RV/Athens. But for technical reasons beyond the contractor's and NCSC's control sufficient hydrodynamic data has not been available. Hence in coordination with NCSC the contractor has proceeded to analyze the near ship bubble transport and source problem considering that the needed data could be input in principle.

The contractor has reviewed the literature on near ship and particularly near stern ship hydrodynamics and surface ship and sea state bubble populations. This review has revealed the existence of established bubble sources and flow zones and the necessity of modifying the bubble evolution equation to include the special effects of near ship hydrodynamics. The bubble population could be predicted in principle on a suitable initial data surface, given sufficient data on the near ship hydrodynamic flow. These topics are reviewed in the following section.

The modification of the bubble evolution equation takes place in section III. The original bubble evolution equation employed simulated the turbulent diffusion of bubbles through use of a single diffusion coefficient and resulted in a 2nd order differential equation for the bubble evolution. However reflection shows that this is not the most general form an evolution equation could take - the current form used is an adaption of a 1d + time approach reported earlier. Hence the modification (or upgrading) of the bubble evolution equation is carried out in 3 spatial dimensions + time using the notion that the hydrodynamic flow can be represented by a random walk.

The modification of the bubble evolution equation leads to the existence of antisymmetric and symmetric bubble transport coefficients. The physical identification of these transport coefficients is carried out in section IV.

The form of the bubble distribution function is discussed in section V. The general form of the bubble distribution function is derived from considerations of bubble dynamics and experimental data. Procedures are then suggested for simplification of the bubble wake evolution modelling.

In section VI the computer programming of the additional terms introduced into the bubble evolution equation is discussed and finally the import of the work is summarized.

II. Surface Ship Bubble Sources and the IDP

A literature review indicates the existance of 4 possible classes of bubble sources for a surface ship's bubble wake. These include an ambient bubble population, air entrainment along the ships hull boundary layer, air entrained in the separated retarded region at the ship's stern, and bow and stern breaking waves. For each of these mechanisms one must have a mechanism for distributing the bubbles from these sources into the ship's hydrodynamic wake.

IIA. The Bubble Sources

The ambient bubble population has been characterized as having an approximate universal character^1. A log-log plot of this distribution (bubble density against radius) is displayed in Figure I, with the height of the curve being a function of wind speed alone for large fetches. Figure's II- IV^2-4 illustrate flow mechanisms in the neighborhood of a ship's hull. Their purpose is to demonstrate that near surface sea state bubbles are certainly brought down to keel depths by nearly streamlined flow approximately following the ship's buttock lines and by vortex action resulting from bow, bilge, and stern vortices. It is known that at the higher frequencies the ambient sea state attenuations are comparable to those of observed wake attenuations. Furthermore, measurements made at sea with an optical bubble counter placed ahead of a ship's propeller show a noticable dependence upon sea state^5, while the observed distribution in radius nearly coincides with the sea state distribution.

Air entrainment along a ship's hull provides another important source for the wake bubble density. Sidney Express measurements, again with the bubble counter ahead of the propeller, indicate a strong fluctuation in the optical bubble count with ship pitch in heavy seas. This depends in addition strongly upon the ship's orientation to the seaway^5. The capture mechanism here is undoubtedly turbulence and vorticity present along the ship's boundary layer.

The remaining two sources are easily recognized from visual observations. These include bubbles introduced by breaking bow and stern waves and at the flow separation region astern the vessel. In the latter case entrainment of bubbles into the wake occurs in the same way as at the hull boundary layer, with the exception that here the boundary interface between the separated-retarding flow region and the vorticiy diffusion zone plays the role of the boundary layer-streamline flow interface (Figure III) along the ship's hull. The capture mechanism is again vorticity, swirl, and turbulance along this interface. These flow regions have been observed experimentally and calculated theoretically^6,7

With the breaking bow and stern waves as sources the capture mechanism is the usual vorticity and swirl associated with the physics of breaking waves, along with the previous mentioned bilge and stern vortices. It should be pointed out here that the bow and stern waves can also combine with ambient chop to produce breaking waves away from the ship even when the bow and stern waves are in regions where they are not breaking. However, in these regions one can assume that the wave vortices themselves are not of sufficient strength to entrain the bubbles into the deep wake and that this region is away from the regions where the bilge and stern vortices play an important role. Thus the latter can be expected to contribute to a shallow wake of extended width but of depth less that the keel depth.

In the schemes discussed the propeller itself has not played an important role as a source of bubbles. However, it is known to shift the position of the separated-retarded flow zone interface^6 , to produce an intake from which it pumps bubbles from the above sources into the deep wake, and introduces additional swirl and vorticity for the capture and distribution of bubbles from the above sources into the deep wake.

Given the sources of the surface ship bubble wake as described, it seems reasonable to specify initial data (for the initial value problem) for the bubble density evolution equation along the sea surface and flow zone interfaces. The mathematical basis for setting up the initial value problem is described in the next section.

IIB. Sources and the Bubble Boundary Value Problem

The mathematical illustration of these notions is most simply carried out using a Green's function analysis to set up the boundary value problem for the bubble evolution equation. The steady state bubble evolution equation (whose derivation will be carried out in the next section) is

$$\vec{\nabla} \cdot \{ \vec{u} \Psi + \vec{J} \} + \{ q \Psi \}_{r} = 0. \tag{1}$$

Here u is the average bubble velocity, Ψ the bubble density , q the bubble disolution rate, while

$$J^{l} = D^{lm} \nabla_m \Psi + G^{lm} \nabla_m \Psi \tag{2}$$

characterizes the effect of velocity fluctuations from the average. The tensors D^{lm} and G^{lm} characterize "diffusion" associated with deviations from the average due to conventional diffusion and vorticity. Their specific forms are given below.

Using Green's theorem it is not difficult to see that the solution to the boundary value problem is specified by the integral

$$\Psi(\vec{x}, r) = \int_S dS^j \int dr' \{ \phi u^j \Psi + \phi G^{jk} \nabla_k \Psi + \Psi G^{jk} \nabla_k \phi - \phi D^{jk} \nabla_k \Psi + \Psi D^{jk} \nabla_k \phi \}, \tag{3}$$

where S is a closed 2d spatial surface, (x,y,z) represents an interior point,and ϕ represents the Green's function belonging to (1), a linear equation.

The surface S can be identified with that discussed in the last section and illustrated in Figure VI if there are no very deep bubbles and sufficiently far from the wake axis ambient bubble transport into the wake can be neglected. Then what the Green's function analysis shows is that specification of the bubble density on some cross-plane and along a fluid interface (such as along part of the ships boundary layer and the boundary between the separated and near streamline flow regions) is sufficient to determine the bubble distribution at the interior point. An alternate proceedure would be to to place a cross plane ahead of the ship and treat the remaining part of S as the sea surface and the boundary of the boundary layer running along the ships hull and the separation zone. This latter situation is illustrated in Figure VII. In general one needs specification of the bubble density on some cross-plane normal to the ship's axis and along some fluid boundary surface, which may be a interface between different well defined flow regions (such as defined by the flow zones of Figure XX) along which the bubble density can be surmized.

III. Modification of the Bubble Evolution Model

The expression for the upgraded bubble transport and convection model follow from viewing the general fluid motion as a biased three dimensional "random walk". Here the fluid flow field is associated with a probability p_n and considered as approximately streamlined for a duration equal to a "correlation time" t . Thus each probable fluid trajectory bringing bubbles to a spacial point x is associated with a probability p_n and a velocity v_n and belongs to the evolution equation

(4) $$\psi_t^{(n)} + \vec{\nabla} \cdot \{ \vec{v}_n \psi^{(n)} \} + \{ g \psi^{(n)} \}_r = 0 .$$

The solution for each n then follows from the method of characteristics:

(5) $$\psi^{(n)}(\vec{x}, r, t) = \psi(\vec{x}_n, r_n, 0)\, e^{-\int_0^t g_n(t')\, dt'} ,$$

where the map between $\vec{x}, r, t$ and $\vec{x}_n, \vec{r}_n, \theta$ is defined by the ordinary differential equations

(6) $$dt = \frac{dr}{g} = \frac{dx^i}{v_n^i} = -\frac{d\psi}{g_n \psi} .$$

IIIA. Application of the 3d Random Walk Process

Averaging the set (4) over n one finds

$$\vec{\nabla} \cdot \{\Sigma p_n v_n \psi^{(n)}\} + \{\Sigma p_n g \psi^{(n)}\}_r = 0. \tag{7}$$

Here it is assumed that we are operating in the coordinate system fixed with the ship and that the wake system is in steady state. Then

$$\{\Sigma p_n \psi^{(n)}\}_t = 0. \tag{8}$$

Defining

$$\Delta \vec{v}_n = \vec{v}_n - \vec{u}, \qquad \psi = \Sigma p_n \psi^{(n)}, \tag{9}$$

where

$$\vec{u} = \Sigma p_n \vec{v}_n \tag{10}$$

is the average bubble velocity at (x,r), and using the inverse of (4) one obtains

$$\vec{\nabla} \cdot \{\vec{u}\psi + \Sigma p_n \Delta \vec{v}_n e^{I_n} \psi(\vec{x}_n, \vec{r}_n)\} + (g\psi)_r = 0. \tag{11}$$

Now $\psi(\vec{x}_n, \vec{r}_n)$ can be related to the average density at $(\vec{x}, r)$ by using the first 2 terms of a Taylor series expansion. This means that application of the analysis cannot be carried across flow zone boundries, where gradients may be large. Using such an expansion one obtains

(12) $$\vec{\nabla}\cdot\left\{\vec{u}\Psi + \sum p_n e^{I_n}\Delta\vec{v}_n\Psi - \sum p_n e^{I_n}\Delta\vec{v}_n\,(\Delta\vec{X}_n\cdot\vec{\nabla}\Psi) - \sum p_n e^{I_n}\Delta\vec{v}_n\,\Delta r_n\Psi_r\right\} + (g\Psi)_r = 0,$$

were $\Delta\vec{X}_n$ and $\Delta\bar{r}_n$ describe the net spatial and radial displacements along the nth bubble trajectory. This form for the bubble density evolution equation can be simplified further by making some observations about the approximate nature of $\Delta\vec{X}_n$, Δr_n, and ΔI_n.

IIIB. Trajectory Assumptions

Let $\vec{u}_F$ denote the average fluid velocity at $\vec{X}$ and $\bar{u}_T$ the bubble buoyant terminal velocity corresponding to the radius r, so that

(13) $$\vec{u} = \vec{u}_F + \vec{u}_T\,.$$

Note that the relative acceleration of the bubbles relative to the fluid is ignored (studies show that the bubbles and time scales of interest are such that a bubble can be considered to reach its terminal velocity almost instantaneously)[11].

Let $\vec{x}_n$ refer to the startpoint of each bubble trajectory terminating at $\vec{X}$. The net vector displacement of the bubble, $\vec{X} - \vec{x}_n$, can be considered as the vector sum of the net displacement along the fluid streamline originating at $\vec{x}_n$, $\Delta\vec{S}_n$, and the vector displacement

$$b_n = \int_0^t u_T(t')\,dt' \tag{14}$$

associated with the vertical buoyant rise. This can be expected to be approximately the case if the correlation time t over which each of the equations (4) is assumed valid is small and if the length scale defining the flow velocity gradients are large compared to the relative distance a bubble rises over the correlation time. This means that the analysis is restricted to near the ship and cannot cross sharp flow boundaries.

In this approximation all the possible fluid streamlines so defined can be assumed to terminate at the same point $\vec{X}'$ below $\vec{X}$. Here we also have the additional assumption that the mean flow dominates to the extent that the bubbles arriving at $\vec{X}$ with radius r all have essentially the same decay history. Thus under the assumption that the mean flow dominates one can associate the change in the bubble radius along each trajectory with that on the mean trajectory. The net displacement of a bubble along its nth trajectory can then be described by the equation

$$\Delta\vec{X}_n = \Delta\vec{S}_n + \vec{b} \tag{15}$$

where

$$\Delta\vec{S}_n = \int_0^t u_F(t')\,dt', \tag{16}$$

and $\vec{u}_F(t')$ is the velocity along the fluid streamline passing through x .

IIIC. The Transport Coefficients

Since it is assumed that

(17) $\Delta r_n = \Delta r \, , \qquad I_n = I \, ,$

one finds to 1st order the steady state equation (in component notation)

(18) $\nabla_\ell \left\{ u^\ell \psi - e^{I} \sum p_n \Delta v_n^{\ell} \Delta s_n^{m} \nabla_m \psi \right\} + (g\psi)_r = 0 .$

Comparing this equation with (1) and (2) one finds that

(19) $D^{\ell m} = e^{I} \sum p_n \Delta v_n^{(\ell} \Delta s_n^{m)} \, , \qquad G^{\ell m} = e^{I} \sum p_n \Delta v_n^{[\ell} \Delta s_n^{m]} \, ,$

where ( ) and [ ] bracketing the exponents denotes tensor symmetrization and antisymmetrization.

IV. Physical Interpretation of the Transport Coefficients

The bubble transport coefficients can be related to physically identifiable parameters.

IVA. Vorticity Induced Transport

Let

(20) $$\Delta\vec{s}_n = \overline{\Delta\vec{s}} + \delta\vec{s}_n \,, \qquad \Delta s = \Sigma p_n \Delta\vec{s}_n .$$

Since one also has from (19) that

(21) $$G^{lm} = e^{I} \Sigma p_n \, \Delta v_n^{[l} \, \delta s_n^{m]} ,$$

one readily sees from the definition of angular momentum that

(22) $$G^{lm} = \frac{2}{\rho} e^{I} \Delta L^{lm}$$

where ΔL^{lm} is the average relative the angular momentum of the fluid elements reaching x' during the correlation time and ρ is the density of the fluid.

Referring to Figure VIII, one sees that G^{lm} can be non vanishing only if there is curvature in the streamline. Writing

(23) $$\Delta v_n^{\,l} = w_n^{lk} \delta s_n^{k} ,$$

one sees that

(24) $$G^{\ell m} = \frac{1}{2} e^{I} \Sigma p_n \left(W_n^{\ell q} \delta S_n^{q} \delta S_n^{m} - W_n^{mq} \delta S_n^{q} \delta S_n^{\ell} \right).$$

Letting

(25) $$W_n^{\ell m} = \omega^{\ell m} + \Delta W_n^{\ell m}, \qquad \omega^{\ell m} = \Sigma p_n W_n^{\ell m},$$

one sees to first order that

(26) $$G^{\ell m} = \frac{1}{2} e^{I} \omega^{\ell k} \Sigma p_n \delta S_n^{k} \delta S_n^{m} - \frac{1}{2} e^{I} \omega^{\quad k} \Sigma p_n \delta S_n^{k} \delta S_n^{\ell}$$

But then $G^{\ell m}$ can be expressed in terms of the Reynold's stress:

(27) $$G^{\ell m} = -\frac{t^2}{2} e^{I} \left\{ \omega^{\ell k} \tau^{km} - \omega^{mk} \tau^{k\ell} \right\},$$

where[8]

(28) $$\tau^{mn} = -\overline{u^m u^n},$$

and where again t is the correlation time over which the displacements take place.

IVB. The Diffusive Transport Coefficient

The diffusion term can also be related to the Reynolds stress. Using (19),

(29) $$D^{\ell m} = t e^{I} \Sigma p_n a_n^{(\ell} \Delta S_n^{m)},$$

where $\vec{a}_n$ is the relative average acceleration over the time t. Hence, the trace ΣD^{ll} is proportional to the average work done, the change in relative kinetic energy. Since the initial relative kinetic energy is zero, as follows from the method of construction of the trajectories, this average work done is just the average fluctuating kinetic energy. However the latter coincides with the negative of the trace of the Reynold's stress tensor. Hence, on the basis that the 2 tensors have the same trace we assert

$$D^{lm} = -\frac{e^{I}}{2} t \, \underset{-}{\tau}^{lm}. \tag{30}$$

IVC. The Characteristic Scaling Function

The exponent I in (7) measures the change in scale due to the change in the "volume" elements associated with the transformation of (x,y,z;r) into $(x_n, y_n, z_n; r_n)$ and involves an integration along the average trajectory. Since the time scale t in the near wake region is short this integration can be approximated by a linearized transformation defined by the mean trajectory for bubbles of radius r reaching X. But this characteristic scaling function is already calculated in the NCSC "Random Walk" program^9, that is the computer routine for I along a trajectory can be lifted from this program.

IVD. The Correlation Time

The correlation time cannot be estimated from the above analysis. But the random walk concept can be used to estimate its numerical value.

Let D represent the magnitude of a diffusion coefficient characterizing a random walk. Then the effective diffusive displacement after a time t is given by

$$X_D = \sqrt{Dt} \quad (31)$$

On the other hand the mean flow has traversed a distance ut, where u is the average mean flow velocity over the elapsed time.

The ratio

$$\frac{\sqrt{Dt}}{ut} \quad (32)$$

defines the "slope" characterizing a diffusive change in the fluid trajectory. When this slope is equal to that given by

$$t\,|\nabla u| \quad (33)$$

which defines a slope characterizing changes in the convective velocity then one can say that the flow is no longer correlated. Hence the correlation time for the turbulent "random walk" flow is given by

$$\frac{\sqrt{Dt}}{ut} = t\,|\nabla u| . \quad (34)$$

Taking for D trace of the symmetric bubble diffusion coefficient (30) one obtains the correlation time

(35)
$$t = e^{I/2} \frac{\sqrt{\tau^{u}}}{|u| \cdot |\nabla u|} .$$

V. The Bubble Density Distribution Function

It is stated in the "Statement of Work - Technical Requirements" that the bubble sizes within a small element of volume may be approximated by a sea state distribution, a conclusion derived from observations on oceanic bubble distributions and illustrated in Figure I. A log-log plot of bubble density vs radius suggests such a distribution may be approximated by a maximum value, a power law increase in radius for small radii, and a power law decrease for larger radii. An analytical investigation has been carried out to determine the basis of this observation in order to determine a way to simplify the numerical integration of (1-2) or work instead with a derived equation involving only the 4 parameters.

A result based upon the fixed suspension model most clearly illustrates the "sea state" distribution concept. In this model the effect of turbulence is taken as keeping the "average" bubble at a fixed spatial location but with a non-vanishing relative velocity with respect to the fluid. The appropriated steady state continuity equation is then given by

$$\frac{dr}{g} = -\frac{d\psi}{g_r \psi} . \tag{36}$$

This equation can be integrated directly. The density for bubbles of radius r at time t is then given in terms of the density at time t_0 by

(37) $$\psi(r) = \psi(r_0) \cdot \frac{g(r_0)}{g(r)} ,$$

where r_0 is the radius of the bubble at time t_0.

Thus one sees that the bubble density distribution is "modulated" by a scaling function independent of the initial density and dependent only upon the disolution model. This is the source of the low radii behavior of the sea state bubble distribution, the large radii behavior following from the bubble creation and entrainment process. Figure IX illustrates the effect of the bubble density scaling factor at small radii. The figure is based upon the notion that the bubble radial decay velocity becomes infinite as $r \to 0$. Thus given $\psi(r_0) = 1$, $\psi(r)$ = 0 even after an arbitrarily short time interval. The maximum in the curve is produced by a minimum in the bubble decay velocity. This together with observations by Wu^1 suggests that the Wake bubble disyribution function can be represented by a function of the form

(38) $$\psi(r) = \psi_0 \frac{(r/a)^n}{1 + (r/b)^m} .$$

where n, m, a, and b are 4 parameters which vary slowly as the wake evolves. The consequent possibility of simplifying the wake evolution code is being investigated.

The assumed general form for this free bubble distribution function can be used as initial data along the flow boundaries specifying the bubble boundary value problem.

VI. Computer Programming Changes

The programming of the initial data boundary value problem awaits sufficient knowledge of the near hull and propeller flows and turbulence in the neighborhood of the sea surface. By agreement with the NCSC POC the "programming" here will be restricted to indicating the nature of the programming changes.

The programming for the characteristic scaling function is already contained in a previous NCSC program^9. Therein included in subroutine "char" the scaling function is calculated under the name "decay factor". The code need only be adapted to the mean trajectory described above, instead of the characteristic curves in the "random walk" program^9.

The antisymmetric and symmetric bubble transport coefficients can collectively replace the present diffusion tensor already included in the NCSC "BW3D"^10, or the more recent upgrade of that code. Effectively, as far as the programming is concerned one in effect just has a new diffusion tensor, but as discussed in the following summary the antisymmetric transport coefficient will bring in some important new dynamics into the modelling.

VII. Summary

From the discussion on the bubble sources and the initial data problem, one sees that an IDP is insufficient for a determination of the bubble evolution. Rather one has mathematically a boundary value problem. The choice of the boundary value surface on which the "initial" data must be determined for a speficiation of the subsurface bubble density is not trivial. The analysis shows that the original concept of a near stern IDP is insufficient for a rigorous solution to the evolution problem since without knowledge of the hydrodynamic flow around the ship the required distribution on this plane cannot be determined. Rather the initial data should be the sea state distribution on a forward cross-plane normal to the ships axis and some specification of the bubble density along the sea surface and the flow boundary layer along the hull and separation zone, as depicted in Figure VII. The literature review shows that the hydrodynamics of the flow around a hull is not an intractable problem^4.

Viewing the problem as one of steady state the bubble density must be specified along the sea surface and the interface separating the streamline zone from the boundary layer. This can be done in analogy to the manner in which the white water foam thickness is determined by the subsurface sea state bubble current^12.

The modifications to the bubble evolution equation involved the bubble transport current. The ordinary diffusion tensor was improved and a new term involving vorticity was introduced. It is important to note that the latter does not contribute to the 2nd order derivative term in the evolution equation, a consequence of its antisymmetry. However, it does introduce a new physical effect in that it provides for bubble transport away from the boundary value surface, the contribution depending upon the wake vorticity through (27). Within the equation of motion one sees that the associated net bubble current is determined by gradients of the vorticity contributes to the bubble current more in the way of a convective term. Thus one has an additional mechanism for taking bubbles off a zonal interface or down from the sea surface - the turbulence at such a surface allows swirl to capture bubbles which would normally not leave the surface.

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FIGURE I

AMBIENT SEA STATE PROBABILITY DISTRIBUTION

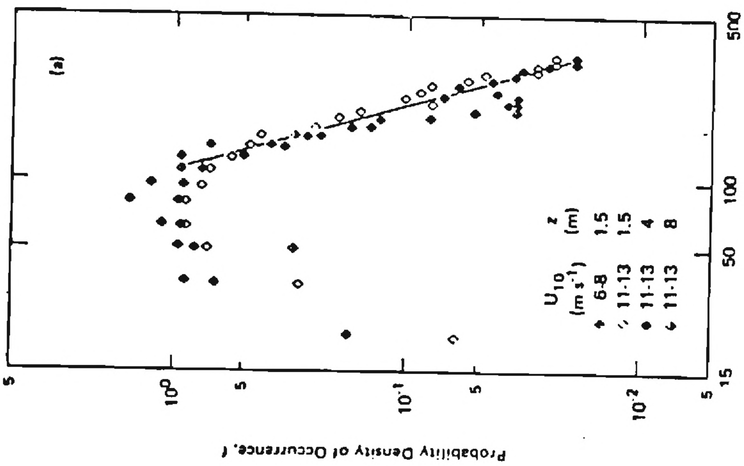


FIGURE II

STREAMLINES ALONG SHIP'S HULL

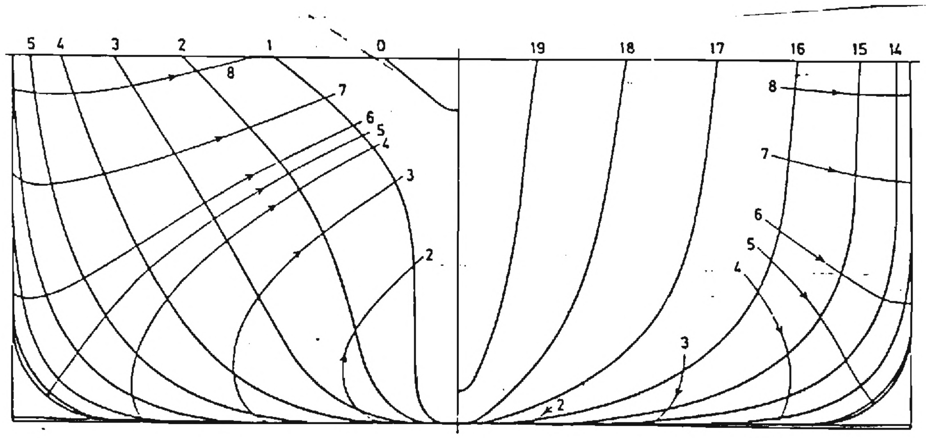
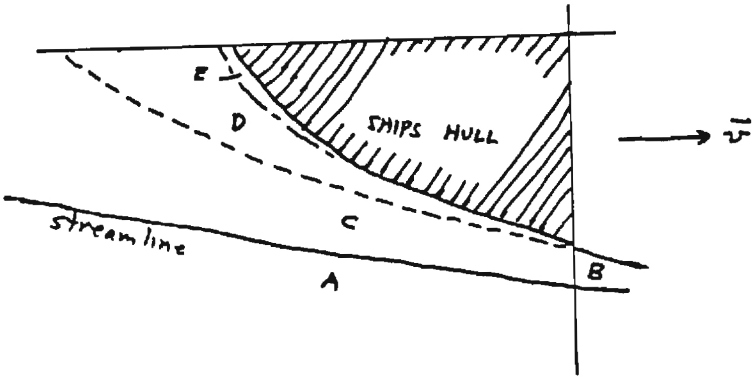


Fig 8. Model 720 SSPA. Calculated streamlines

FIGURE III

FLOW ZONES NEAR SHIP'S STERN



A: Potential Flow Region

B: Boundary layer

C: Vorticity Diffusion Region

D: Separated Retarding Region

E: Viscous Sublayer

FIGURE IV

BILGE AND STERN VORTICES

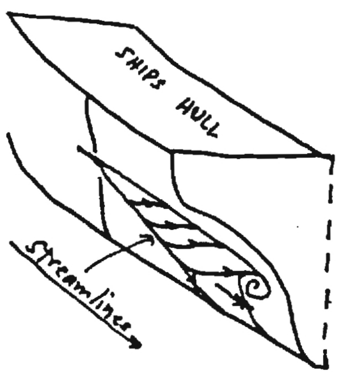


FIGURE V

PROPELLER EFFECTS

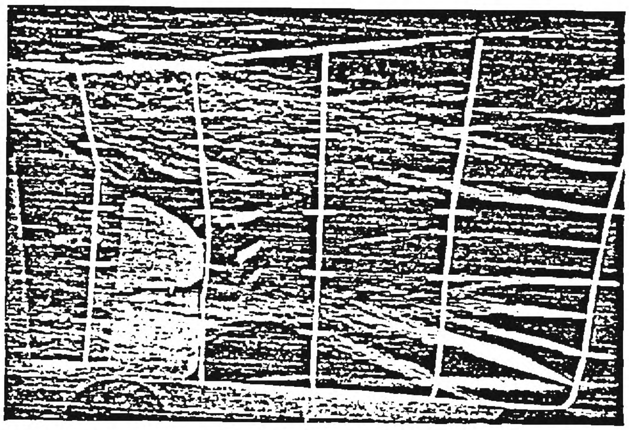


FIGURE VI

THE INITIAL DATA SURFACE (STERN)

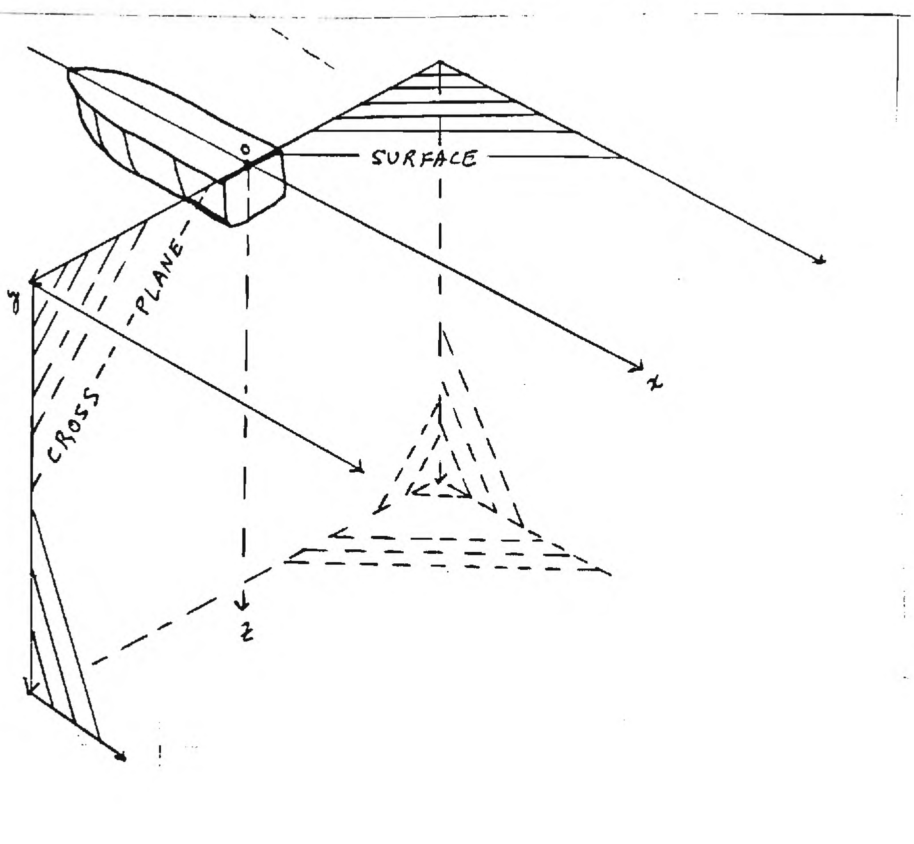


FIGURE VII

THE INITIAL DATA SURFACE (FORWARD)

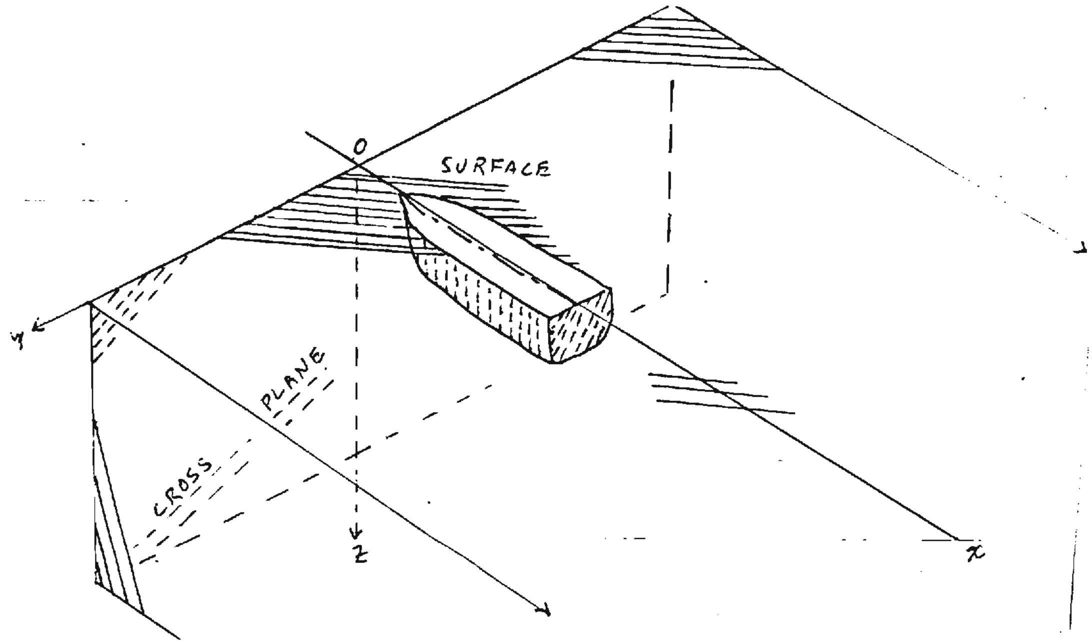


FIGURE VIII

STREAMLINE CURVATURE AND ANGULAR MOMENTUM